Coordinate Geometry

Important Formulas

1) Distance Formula: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
2) Midpoint Formula: $\text{midpoint} = \left(\frac{x_2 + x_1}{2}, \frac{y_2 + y_1}{2} \right)$
3) Slope Formula: $m = \frac{y_2 - y_1}{x_2 - x_1}$

Equation of a Line

1) Slope Intercept Form:

$y = mx + b$, where m = slope and b = y-intercept

2) Point Slope Form:

$y - y_1 = m(x - x_1)$, where (x_1, y_1) = a point on the line and m = slope

Parallel and Perpendicular Lines

1) Parallel Lines in a Coordinate Plane: In a coordinate plane, two non-vertical lines are parallel if and only if they have the **same slope**.
2) Perpendicular Lines in a Coordinate Plane: In a coordinate plane, two non-vertical lines are perpendicular if and only if the product of their slopes is -1. In other words, the two slopes must be **opposite (opposite signs) reciprocals**.

Example 1) a) Find length of $\overline{AB}$. b) Find the midpoint of $\overline{AB}$

a) Length of AB

$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ (Use the Distance Formula)

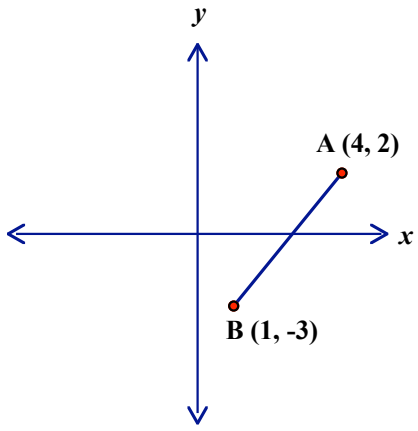
$= \sqrt{(4-1)^2 + (2-(-3))^2}$

$= \sqrt{(4-1)^2 + (2+3)^2}$

$= \sqrt{(3)^2 + (5)^2}$

$= \sqrt{9 + 25}$

$= \sqrt{33}$



b) Find the midpoint of AB

$\text{midpoint} = \left(\frac{x_2 + x_1}{2}, \frac{y_2 + y_1}{2} \right)$ (Use the Midpoint Formula)

$= \left(\frac{4+1}{2}, \frac{2+(-3)}{2} \right)$

$= \left(\frac{5}{2}, -\frac{1}{2} \right)$

Example 2) Line r passes through (-2,2) and (5,8). Line s passes through (-8,7) and (-2,0). Is r ⊥ s? (Use the slope formula)

How do I approach this problem? *We are looking to see if the slope of line r, m_r, and the slope of line s, m_s, are opposite reciprocals.*

Use the slope formula to find the slope of each line.

$$m_r = \frac{y_2 - y_1}{x_2 - x_1} = \frac{8-2}{5-(-2)} = \boxed{\frac{6}{7}}$$

$$m_s = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0-7}{-2-(-8)} = \boxed{-\frac{7}{6}}$$

What does this mean?

Notice that $m_r = \frac{6}{7}$ and $m_s = -\frac{7}{6}$ are opposite reciprocals. Therefore, <u>the two lines are perpendicular.</u>

Example 3) An equation for line v is $y = -\frac{3}{2}x + 5$. An equation for line w is $6x + 4y = 7$. Is $v \parallel w$?

How do I approach this problem? *We need to compare the slope, m, of each line. Put the second equation in the form y = mx + b so that we can identify the slope, m.*

Line *w*

$6x + 4y = 7$

$4y = -6x + 7$

$y = -\frac{3}{2}x + \frac{7}{4}$

$m_w = -\frac{3}{2}$

Line *v*

$y = -\frac{3}{2}x + 5$

$m_v = -\frac{3}{2}$

What does this mean?

<u>Both lines are parallel</u> because the slope of line *w* and the slope of line *v* are the same. Both slopes are $-\frac{3}{2}$.