Isosceles, Equilateral, and Right Triangles

Isosceles Triangles
In an isosceles triangle, the angles across from the congruent sides are congruent. Also the sides across from congruent angles are congruent.

Example 1) Find the value of x and y.

Solution:
Since triangle BDC is isosceles, then the angles opposite the congruent sides are congruent. Therefore, \( x = 70^\circ \)

The sides opposite the 32º angles are congruent. Therefore, \( y = 10 \).

Equilateral Triangles
In any equilateral triangle, all sides are congruent and all angles are congruent. The measure of each angle is 60º.

Right Triangles
The Pythagorean Theorem helps you find the side lengths of right triangles. For a more extensive explanation, see the “Pythagorean Theorem” link on the SI PSAT website.

Example: Find the BC

Pythagorean Theorem
\[ a^2 + b^2 = c^2 \]
where \( a \) and \( b \) are the legs and \( c \) is the hypotenuse.

\[
egin{align*}
(BC)^2 & = 85 \\
BC &= \sqrt{85}
\end{align*}
\]
Special Right Triangles

45º-45º-90º Triangle
In a 45º-45º-90º triangle, the hypotenuse is $\sqrt{2}$ times as long as each leg.

30º-60º-90º Triangle
In a 30º-60º-90º triangle, the hypotenuse is twice as long as the shorter leg, and the longer leg is $\sqrt{3}$ times as long as the shorter leg.

Example:
1) Find the missing side length.

Solution:
Since the ratio of a 45-45-90 triangle is 1:1: $\sqrt{2}$ then the leg across from the 45º angle is 4 and the hypotenuse is $4\sqrt{2}$.

2) Find the side of a square whose diagonal is 5 ft.

Solution: sketch a picture
Since the ratio of a 45-45-90 triangle is 1:1: $\sqrt{2}$ and we are given the hypotenuse, 5, then to find the side of the square, we can set up the equation

$$x\sqrt{2} = 5$$

$$x = \frac{5}{\sqrt{2}}$$

The side of the square is $\frac{5}{\sqrt{2}}$. 
3) Find the missing side length.

\[
\text{Solution:}
\]

Since the ratio of a 30-60-90 triangle is \(1: \sqrt{3}: 2\) and we are given the hypotenuse, 16, then the side length across from the 30º angle is 8 and side length across from the 60º angle is \(8 \sqrt{3}\).

4) Find the side length of an equilateral triangle whose altitude is 7 inches.

**Solution:** Sketch a picture

Note: the interior angles of an equilateral triangle are each 60º. When drawing the altitude, you produce a 30-60-90 triangle. Since the ratio of a 30-60-90 triangle is \(1: \sqrt{3}: 2\) and we are given the side across from the 60º angle, 7, then to find the length across from the 30º angle, we can set up the equation

\[
x \sqrt{3} = 7
\]

\[
x = \frac{7}{\sqrt{3}}
\]

This means that the side length across from the 90º angle is \(\frac{14}{\sqrt{3}}\).