

Part 1: Radioactivity (Step 3)

1. An American atomic bomb destroyed the Japanese city of Hiroshima on August 6, 1945, killing approximately 140,000 people.¹ The energy released by the exploding bomb was in the neighborhood of 5.4×10^{13} J.

As you know, this energy was released through the direct conversion of mass to energy via Einstein's theory. How much mass was converted directly to energy? Express your final answer in *grams*.

$$E = mc^2$$

$$5.4 \times 10^{13} \text{ J} = m(3 \times 10^8 \text{ m/s})^2$$

$$m = 6 \times 10^{-4} \text{ kg} = 0.6 \text{ g}$$

2. A Geiger counter is a device which 'clicks' every time it is struck by a radioactive particle. You've placed your Geiger counter a distance of 2 meters from a radioactive source. You note that it is clicking approximately 50 times per minute. Now you move the Geiger counter to a distance of 10 meters (5 times farther away).

- a. How many clicks should you expect in one minute at the new location? Explain.

Since radioactive emission obeys an inverse-square law, you should get 25 times fewer clicks at 5 times the distance. This means instead of 50 per minute you should get 2 per minute.

- b. Conceptually, why should the number of clicks decrease with greater distance from a radioactive source? Include a sketch in your answer.

The particles that are emitted through radioactive decay travel in every direction (randomly). So just as a Geiger counter takes up less of your "field of view" if it is farther away, it also "intercepts" fewer radioactive particles at a greater distance.

3. What role do the electromagnetic force and the strong nuclear force play in radioactive decay? What about the weak nuclear force?

The strong nuclear force binds nucleons (protons and neutrons) together. The electrical repulsion between protons wants to push the nucleus apart. A nucleus is in a state of tension between these two forces. When an alpha decay occurs, it means that the repulsion of protons overcame the attraction from the strong nuclear force. The weak nuclear force plays a different role: one can think of a neutron as a proton, electron, and neutrino "bound together" by the weak force. Then, when a neutron decays, the weak force gives up the proton, electron and neutrino. This is called beta decay.

¹ http://en.wikipedia.org/wiki/Little_Boy

4. The radioactive element ^{238}U spontaneously decays into the element thorium (Th).
- a. What kind of decay is this (alpha, beta, or gamma)? Include with your answer the complete decay equation.

This is an alpha decay, since $^{238}\text{U} \Rightarrow ^{234}\text{Th} + ^4\text{He}$... note that uranium has atomic number 92 and thorium has atomic number 90, so it gave up two protons (a helium nucleus) and two neutrons along with it.

- b. Thorium is itself not stable – it will beta decay into protactinium (Pa). Write down the decay equation for this process.



Note that Pa has an atomic number of 91, one higher than Th ... this is because a neutron turned into a proton. The atomic weight doesn't change, since protons and neutrons weigh basically the same. An electron and neutrino were emitted.

5. Radioactive bismuth (^{210}Bi) has a half-life of almost exactly 5 days. Let's say today you have a sample of 200 g of this radioactive bismuth. Fill in the table below describing how much ^{210}Bi you will have on the days shown. Each 5 day period means the mass will decrease by $\frac{1}{2}$.

Time elapsed (days)	Mass of sample remaining (g)
0	200 g
5	100 g
10	50 g
25	6.25 g
50	0.195 g

Part 2: Energy Conservation (Step 3)

Please use $g = 10 \text{ m/s}^2$ instead of 9.8 m/s^2 for this part. Neglect friction and air resistance.

1. Your locomotive has a mass of $5 \times 10^5 \text{ kg}$ (500,000 kg) and runs on a flat track – no hills.
 - a. You'd like to get your locomotive from rest to a speed of 30 m/s. How much kinetic energy will it have when it's going that fast? Express your answer in appropriate units.

$$K = \frac{1}{2}mv^2 = \frac{1}{2}(500,000 \text{ kg})(30 \text{ m/s})^2 = 225,000,000 \text{ J} = 2.25 \times 10^8 \text{ J}$$

- b. How much work will your engine have to do to get the locomotive moving this fast?

$$2.25 \times 10^8 \text{ J} \text{ -- work is transferred energy}$$

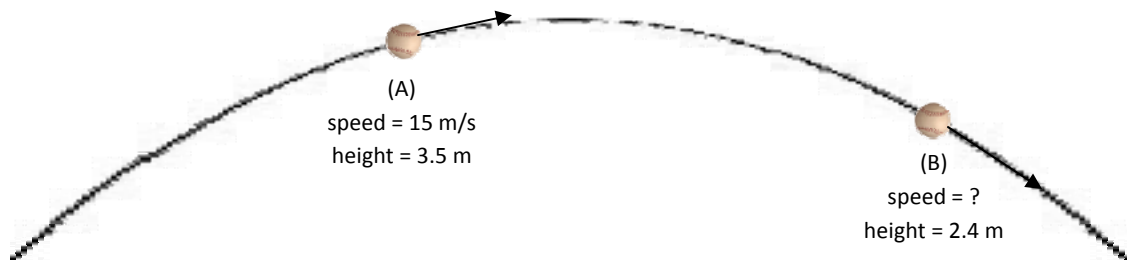
- c. Suppose your locomotive is able to exert a constant force of 300,000 N to push it forward from rest along the track. Starting from rest, how *far* will the locomotive have traveled by the time it achieved a speed of 30 m/s? Express your answer in meters. (*Hint*: this is not a kinematics/Big Three problem; this is a work/energy problem.)

$$W = \text{force} \times \text{distance}$$

$$2.25 \times 10^8 \text{ J} = 300,000 \text{ N} \times \text{distance}$$

$$\text{distance} = 750 \text{ m.}$$

2. You throw a 0.20 kg softball into the air along the trajectory shown below. The height and speed of the softball is shown at point (A). Your job is to determine the speed of the softball at point (B). (Use an energy analysis, not 2D kinematics.)



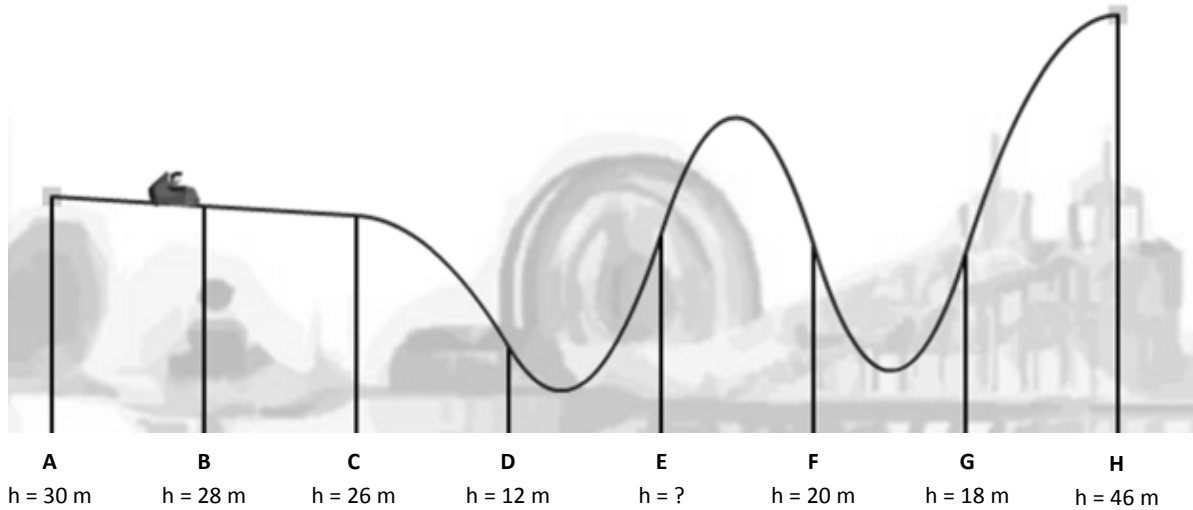
Energy is conserved.

$$E_A = \frac{1}{2}mv^2 + mgh = \frac{1}{2}(0.20 \text{ kg})(15 \text{ m/s})^2 + (0.20 \text{ kg})(10 \text{ m/s}^2)(3.5 \text{ m}) = 29.5 \text{ J}$$

$$E_B = 29.5 \text{ J} = K + mgh = K + (0.20 \text{ kg})(10 \text{ m/s}^2)(2.4 \text{ m}) = K + 4.8 \text{ J} \quad \dots \text{ so } 29.5 \text{ J} = K + 4.8 \text{ J}$$

$$K = 24.7 \text{ J} = \frac{1}{2}mv^2 = \frac{1}{2}(0.20 \text{ kg})v^2 \quad \dots \text{ solve for } v = 15.7 \text{ m/s.}$$

3. Consider the following roller coaster system track. Each point along the track is labeled A – H. The height of the track at most points is shown. (Image is not to accurate scale.)



- a. An empty coaster with mass 120 kg is released from rest at point (A). How fast should the coaster be going at point (D)?

Energy is conserved. $E_A = mgh = (120 \text{ kg})(10 \text{ m/s}^2)(30 \text{ m}) = 36,000 \text{ J}$
 $E_D = 36,000 \text{ J} = K + mgh = K + (120 \text{ kg})(10 \text{ m/s}^2)(12 \text{ m}) = K + 14,400 \text{ J}$
 $K = 36,000 \text{ J} - 14,400 \text{ J} = 21,600 \text{ J} = \frac{1}{2}mv^2 = \frac{1}{2}(120 \text{ kg})v^2 \dots \text{ solve for } \mathbf{v = 19 \text{ m/s.}}$

- b. The speed of the coaster at point (E) is 12.6 m/s. How high is the track at point (E)?

Energy is conserved. $E_A = mgh = (120 \text{ kg})(10 \text{ m/s}^2)(30 \text{ m}) = 36,000 \text{ J}$
 $E_E = 36,000 \text{ J} = \frac{1}{2}mv^2 + mgh = \frac{1}{2}(120 \text{ kg})(12.6 \text{ m/s})^2 + U = 9525.6 \text{ J} + U$
 $U = 36,000 \text{ J} - 9525.6 \text{ J} = 26,474.4 \text{ J} = mgh = (120 \text{ kg})(10 \text{ m/s}^2)h \dots \text{ solve for } \mathbf{h = 22 \text{ m.}}$

- c. You reset the coaster at the start and release it from rest, but this time carrying four passengers weighing a total of 250 kg. How would your answer to part (a) change?

It would not change – the mass cancels out from all simple energy conservation problems like this.

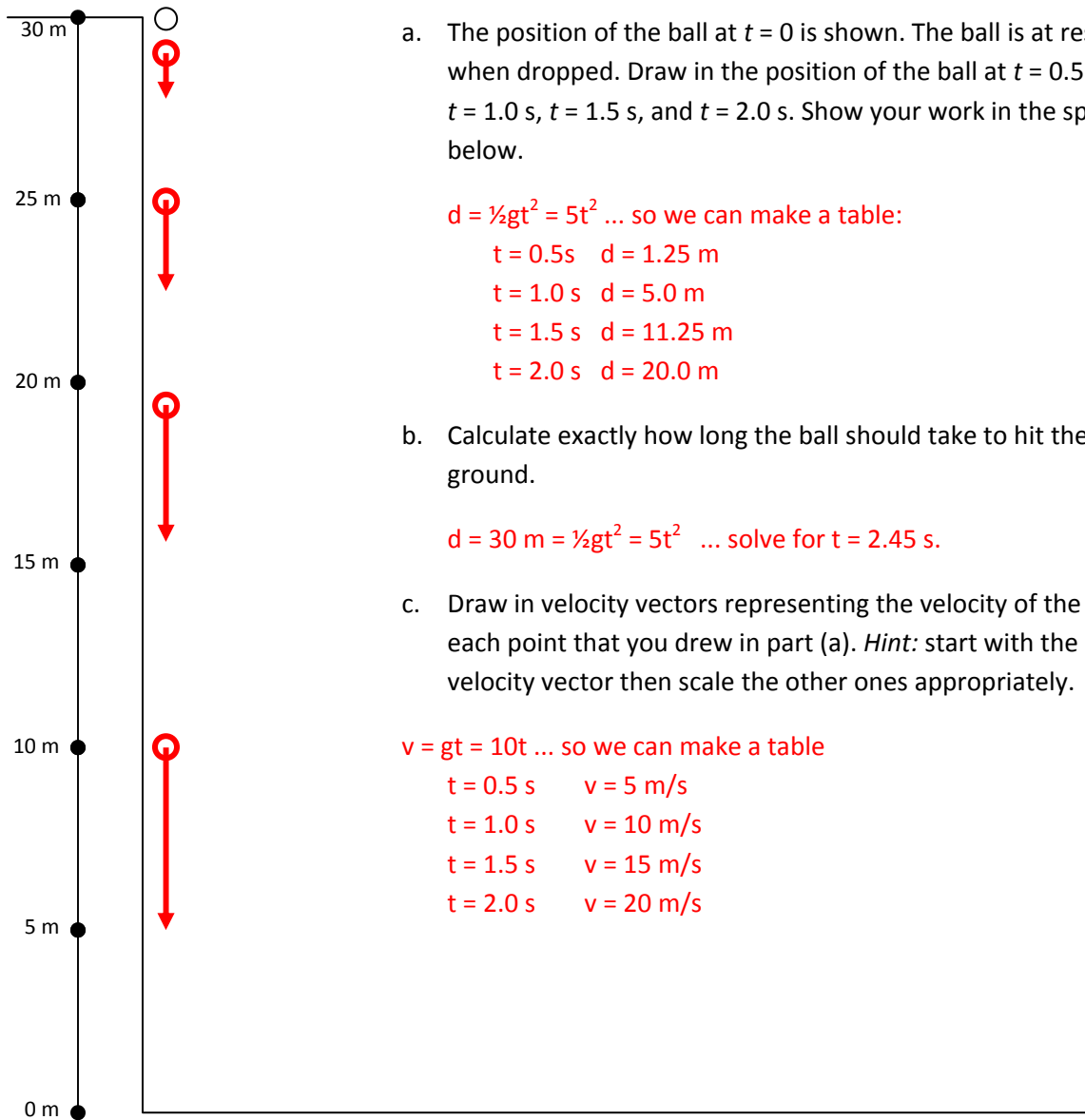
- d. How fast would you have to “launch” the roller coaster at point (A) so that it can *just* make it to the top of the hill at point (H) with zero speed? Express your answer in m/s.

The coaster would need to have $E_H = mgh = (120 \text{ kg})(10 \text{ m/s}^2)(46 \text{ m}) = 55,200 \text{ J}$ of energy to make it to the top of H. It already has 36,000 J of potential at A, so it only needs 55,200 J – 36,000 J = 19,200 J of kinetic at launch to make it. $K = 19,200 \text{ J} = \frac{1}{2}(120 \text{ kg})v^2$ means $\mathbf{v = 17.9 \text{ m/s.}}$

Part 3: 1D Kinematics (Step 2)

Please use $g = 10 \text{ m/s}^2$ instead of 9.8 m/s^2 for this part. Neglect friction and air resistance.

1. You drop a ball from the edge of a 30 m tall cliff. The ball free falls.



- a. The position of the ball at $t = 0$ is shown. The ball is at rest when dropped. Draw in the position of the ball at $t = 0.5 \text{ s}$, $t = 1.0 \text{ s}$, $t = 1.5 \text{ s}$, and $t = 2.0 \text{ s}$. Show your work in the space below.

$d = \frac{1}{2}gt^2 = 5t^2$... so we can make a table:

$t = 0.5 \text{ s}$ $d = 1.25 \text{ m}$

$t = 1.0 \text{ s}$ $d = 5.0 \text{ m}$

$t = 1.5 \text{ s}$ $d = 11.25 \text{ m}$

$t = 2.0 \text{ s}$ $d = 20.0 \text{ m}$

- b. Calculate exactly how long the ball should take to hit the ground.

$d = 30 \text{ m} = \frac{1}{2}gt^2 = 5t^2$... solve for $t = 2.45 \text{ s}$.

- c. Draw in velocity vectors representing the velocity of the ball at each point that you drew in part (a). *Hint:* start with the biggest velocity vector then scale the other ones appropriately.

$v = gt = 10t$... so we can make a table

$t = 0.5 \text{ s}$ $v = 5 \text{ m/s}$

$t = 1.0 \text{ s}$ $v = 10 \text{ m/s}$

$t = 1.5 \text{ s}$ $v = 15 \text{ m/s}$

$t = 2.0 \text{ s}$ $v = 20 \text{ m/s}$

2. If, in the last problem, the ball had been thrown *upwards* with a speed of 5 m/s , but then allowed to fall back down all the way to the ground, how long would it be in the air?

$y(t) = y_0 + v_0t + \frac{1}{2}at^2$... $0 \text{ m} = 30 \text{ m} + (5 \text{ m/s})t - (5 \text{ m/s}^2)t^2$... this is a quadratic equation with solution $t = 3 \text{ seconds}$, since $30 \text{ m} + 15 \text{ m} - 45 \text{ m} = 0 \text{ m}$. (*Note:* this one was a stretch this time!)

3. You are in a race with your little brother. You give him a head start of 10 seconds, and he runs with a constant speed of 4 m/s. When you start running, you simply accelerate at 0.5 m/s^2 until you catch your brother.

a. How long until you catch your brother? Answer in seconds.

Let's say you start running at $t = 0$. Then the position of your brother can be described by the equation

$$x_{\text{brother}} = 40 \text{ m} + (4 \text{ m/s})t \quad \dots \text{ note that he has a 40 m head start.}$$

Your position can be described by

$$x_{\text{you}} = \frac{1}{2} (0.5 \text{ m/s}^2)t^2$$

You catch your brother when these two values are equal to one another:

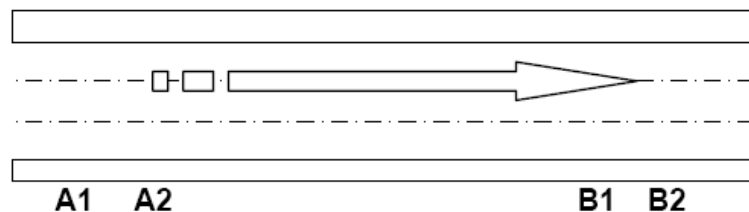
$$40 + 4t = 0.25t^2$$

This is a quadratic equation with solution $t = 23$ seconds.

b. How far from the starting point of the race do you catch up to your brother? Answer in meters.

$$x_{\text{brother}} = 40 \text{ m} + (4 \text{ m/s})t = 40 \text{ m} + (4 \text{ m/s})(23 \text{ s}) = 132 \text{ m.}$$

4. You are out on Sunset Blvd with your buddies measuring the positions of cars at various times.



The distance between point A1 and point A2 is $d_A = 2.5 \text{ m}$. The distance between point B1 and point B2 is $d_B = 1.8 \text{ m}$. The distance between point A2 and point B1 is 18.0 m .

The car passes the four points at the following times:

A1: 12:15:30.010 pm

B1: 12:15:31.575 pm

A2: 12:15:30.135 pm

B2: 12:15:31.935 pm

Calculate the velocity of the car during time interval A1-A2, the velocity of the car during time interval B1-B2, and the acceleration of the car during time interval A1-B1. Include proper units.

$$v_A: \quad \Delta x / \Delta t = 2.5 \text{ m} / 0.125 \text{ s} = 20 \text{ m/s}$$

$$v_B: \quad \Delta x / \Delta t = 1.8 \text{ m} / 0.36 \text{ s} = 5 \text{ m/s}$$

$$a: \quad \Delta v / \Delta t = (5 \text{ m/s} - 20 \text{ m/s}) / 1.565 \text{ s} = -9.6 \text{ m/s}^2 \quad \dots \text{ WOW braking hard!}$$

Was the car **speeding up** or **slowing down**? (Circle one.)