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AP Physics C / 2* / Philhour

DC Circuits / Capacitance Lab

Abstract

The purpose of this lab was ultimately to test the relation between capacitance, electric potential, current, charge, and time and to compare these experiments with our theoretical understanding. This involved making both electric potential and current graphs with respect to time, deriving mathematical expressions to generically represent what these curves should look like, and concluding whether both models agree. Beyond this, we also sought to determine the exact capacitance of a random capacitor to illustrate the overt inaccuracy of the listed value.

I. Introduction

First, let us introduce the physicists themselves (it is a rather stellar group, if we do say so ourselves…):

Max, Derek, Jordan…

and Jared too!

And just look at those handsome, happy campers!

As for the experiment itself, we sought to achieve the highest scientific precision possible. To that end, we conducted our own physical measurements of each material, disregarding any labeled amounts. Therefore, our first step was to measure the resistance of our resistor. Once knowing our resistance, we attached a voltage probe (in parallel) and an ammeter probe (in series) with a capacitor, resistor, and a DC power source tuned to a potential difference of 6 volts (as measured by another voltmeter). Once closing the switch and creating a circuit, we were able to measure the magnitude of the total current and the electric potential across the capacitor as expressed graphically. With this, we could not only draw a curve fit for each graph to examine the accuracy of the data we obtained, but we could also determine the time constant τ for the current versus time graph and mathematically extract the capacitance. This will provide a nice check on the legitimacy of our procedure. We are confident in the accuracy of our data, and hope you are equally convinced as well.

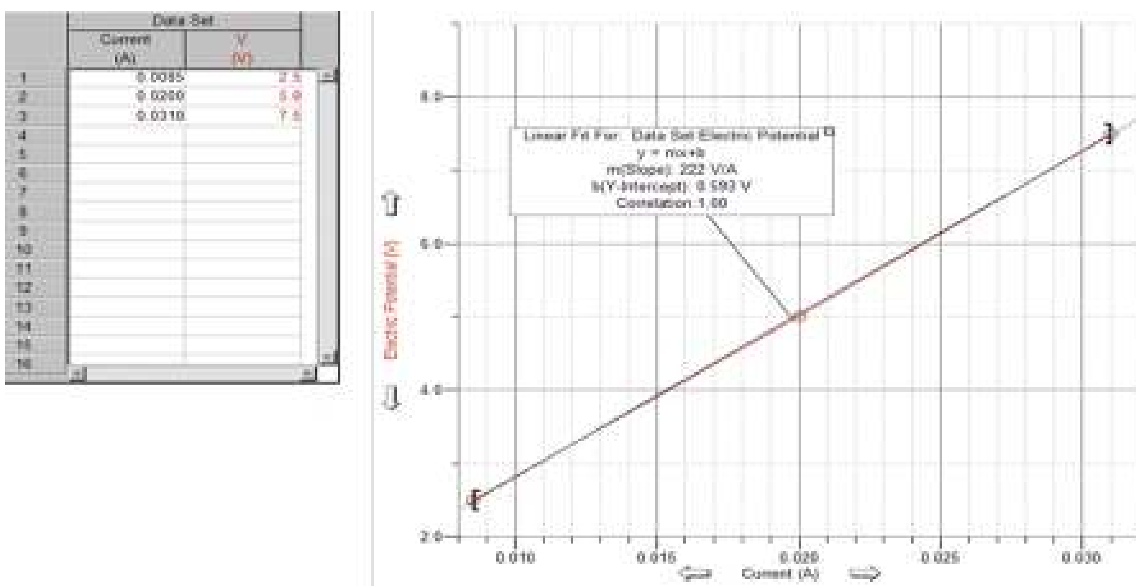
II. Results

Mathematical Models for Current and Potential Through the Capacitor:

We start with the expression that $V_0 – V_1 – V_2 = 0$ (Conservation of Energy).
Using Ohm's Law and the definition of capacitance, $V_0 – IR – (Q/C) = 0$.
Since current is the time derivative of charge, $V_0 – (dQ/dt)R – (Q/C) = 0$.
Multiplying through by C and adding (dQ/dt) to the other side, $CV_0 – Q = RC(dQ/dt)$.
Multiplying through by dt and dividing through by $CV_0 – Q$ and RC, $(dt/RC) = (dQ / CV_0 – Q)$.
Integrating this expression, $t/RC + A = -\ln(CV_0 – Q)$.
Expontentiating both sides, $e^{-t/RC + A} = e^A e^{-t/RC} = CV_0 – Q$.
Therefore, $Q(t) = CV_0 – e^A e^{-t/RC}$.
Since $Q(0) = 0$, $CV_0 – e^A = 0$, and $CV_0 = e^A$.
Thus, $Q(t) = CV_0(1 – e^{-t/RC})$.
Since $Q'(t) = I(t)$, $I(t) = (CV_0/RC)e^{-t/RC} = I_0 e^{-t/RC}$.
Using the definition of capacitance, $V(t) = V_0(1 – e^{-t/RC})$.
Thus, we deduce that the limit as t approaches infinity of $I(t) = 0$, while it equals V_0 (6 V) for V(t).

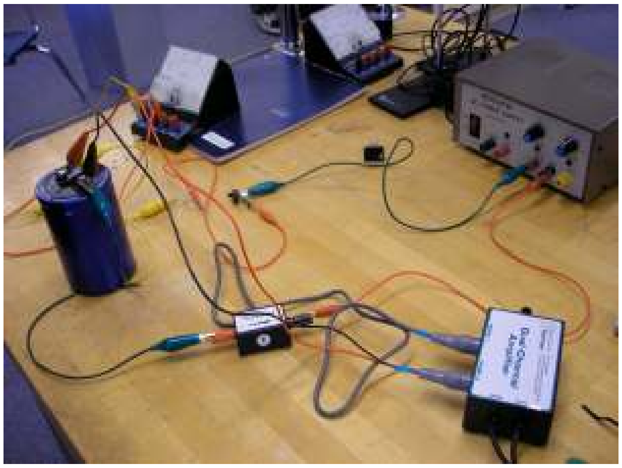
Experimental Data:

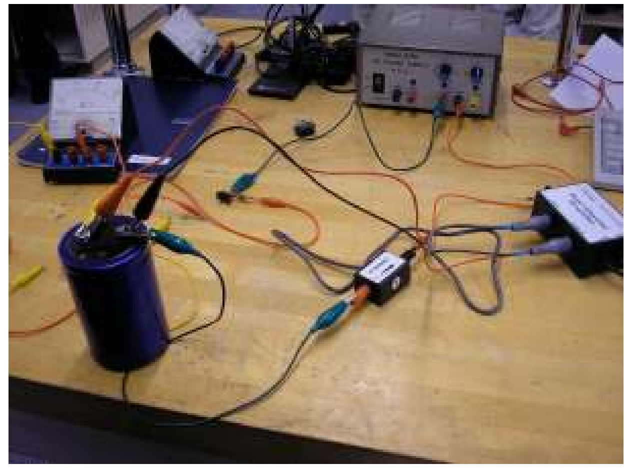
First, we wanted the magnitude of the resistance of our resistor. To that end, we applied three fixed potential differences across it, measured the current able to flow through the resistor at these potentials, and drew a best-fit line, the slope of which represents the resistance.

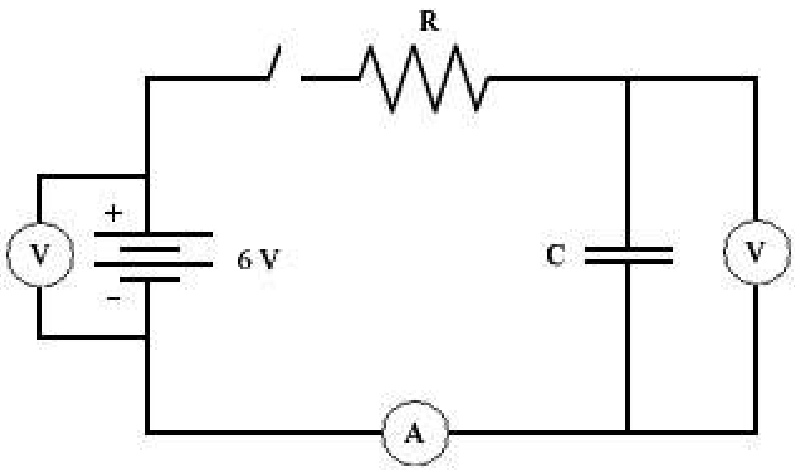


The slope of this line is 222 Ω (V/A), with the linear fit having a 1.00 correlation to our data. Therefore, the symbol R for our equations above corresponds to a numerical value of 222 Ω.

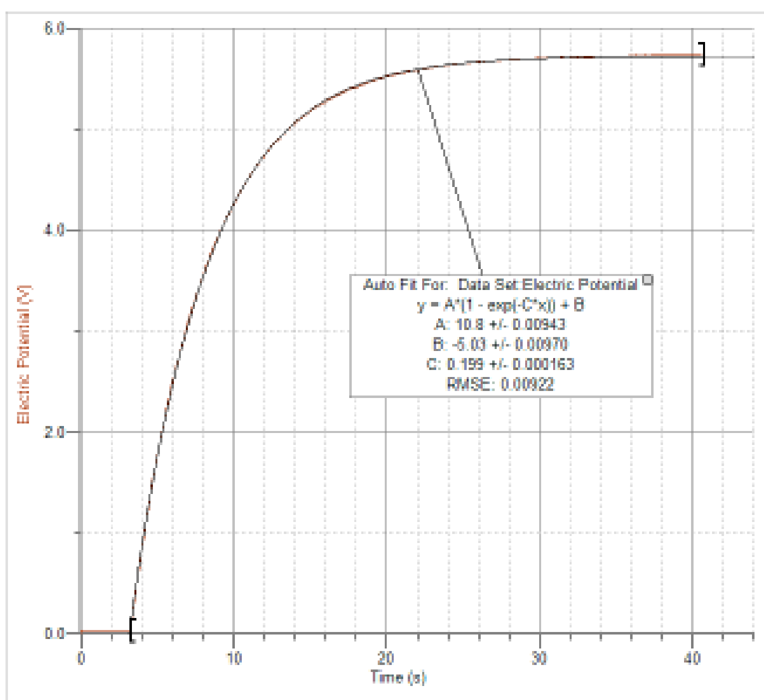
Now to see how our data corresponds to our equations above. Note that, as discussed above, we expect $\lim_{t \to \infty} I(t) = 0$ while $\lim_{t \to \infty} V(t) = V_0 = 6$ V. Another easily observable trait is that $I(t_0) = I_0$, while $V(t_0) = 0$. Our general curve fit will also serve to check our work. Keep in mind that all of this data accompanies the schematic and pictures seen on the following page.





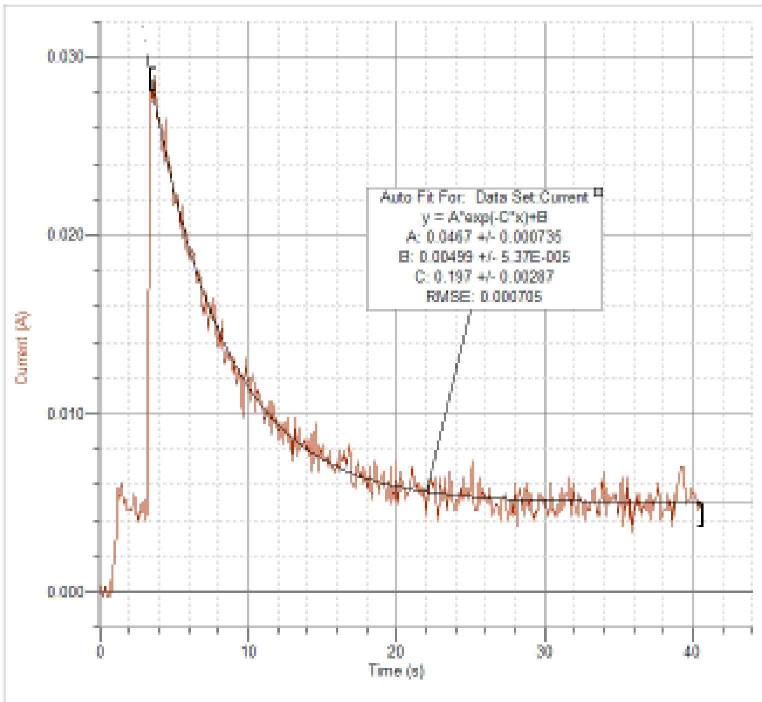


Electric Potential Versus Time:



Notice how exactly the A*(1 – exp(-C*x)) + B curve fit corresponds to our experimental data. This is precisely what we were hoping for! Furthermore, notice that as soon as the switch was closed at $t = t_0$, $V(t_0) = 0$, and that the curve appears to approach the value $V(t) = 6$ V as t increases. This, too, is consistent with our mathematically derived equations.

Current Versus Time:



As for this graph, while there are observable fluctuations in the current values detected by the ammeter probe, the overall curve follows to an astonishing accuracy the exponential decay model deduced previously. This suggests that our theory is viable and applicable. As before, let us check the initial and final conditions. We can see that at $t = t_0$, $I(t_0) = I_0$, while the curve proceeds to zero as t increases, also consistent with our model.

Time Constant (τ) *and Total Capacitance*:
If we define $\tau = t = RC$, then we can see that $I(\tau) = I_0/e$. This directly leads to the expression that $\tau - t_0 = RC$. Based on our data table that coincides with the current versus time graph above, $I_0 \approx .0290$ A, which happens at $t_0 \approx 3.4$ s. This means that $I_0/e \approx .0107$ A, which happens at $\tau \approx 10.8$ s. This means that $C = (\tau - t_0)/R = (7.4\ C/A)/(222\ V/A) = .033$ F (Since $I = dQ/dt$, [A] = [C/s], and [s] = [C/A]; also, since $Q = CV$, [C] = [FV], and [F] = [C/V]).

Quick Check of Our Determined Capacitance:
Because our capacitance differs from the listed capacitance by a factor of 2, we should at least check to see if our value makes sense. If we consider $(Q_{Stored})/(\tau_{RC})$, a quantity that should approximate the current in the circuit at $t = t_0$, note that $Q_{Stored} = CV_0 = (.033\ F)(6\ V)$. Thus, we obtain that $I_0 = (.2\ C)/(7.4\ s) = .027$ A, which is very close to the actual value (.0290 A). Based on this, our capacitance is at least empirically consistent with the data we obtained, leading us to conclude that the value listed on our capacitor was significantly lower than the true capacitance.

III. Sources Of Error

Significant Figures – As with any experiment, our results are only as reliable as the measuring capabilities of our tools. For the sake of scientific precision, we rounded our results regularly to two or three decimal places; however, this could have produced a slight problem in the accuracy of our data. To combat excessive inaccuracy, we only rounded at the end of a particular calculation, leaving the only approximation in the final answer.

Resistance and Temperature – As we have demonstrated, the resistance of an object varies with the temperature of the surrounding environment. Due to a number of trial runs, our resistors may have heated up slightly, in turn altering our measured resistance. This may or may not have affected the resistance that we measured, but if it did, the effect would have been relatively miniscule, as demonstrated by the almost precise correlation between our mathematical models and experimental data.

Discharging the Capacitor – Prior to running the experiment officially, we conducted a few trial tests without the volt probes, just to gain a sense of how we should properly carry out the experiment when actually measuring. When doing so, the capacitor filled to its capacity and the current naturally diminished with time; however, the source of error was introduced when discharging the capacitor following these trials. While we did our best to rid the capacitor of all stored charge via an external light bulb, there might have been some charge left over that could have affected our experiment. Exactly how much this impacted our results is unclear, but we do know that it was miniscule because our graph almost exactly approximates our model's statement that $V(t_0) = 0$.